4  Amplitude Modulation (AM)

4.1  Introduction .............................................. 3

4.2  Double-Sideband Suppressed Carrier AM (DSB-SC) ............... 6
   4.2.1  Modulation .......................................... 6
   4.2.2  Demodulation ....................................... 8
          Pilot Carrier ........................................ 11
          Phase Locked Loop .................................. 14

4.3  Double-Sideband Large Carrier AM ................................ 15
   4.3.1  Modulation .......................................... 15
   4.3.2  Carrier and Sideband Power in AM ....................... 18
   4.3.3  Demodulation ....................................... 19

4.4  Quadrature AM (IQ) ........................................ 20

4.5  Single-Sideband AM (SSB) .................................. 22
   4.5.1  Modulation .......................................... 23
   4.5.2  Demodulation ....................................... 26

4.6  Vestigial-Sideband AM (VSB) ................................ 29
   4.6.1  Video Transmission in Commercial TV Systems ........... 31
4.7 Summary
4.1 Introduction

**Modulation:** Process by which a property or a parameter of a signal is varied in proportion to a second signal.

**Amplitude Modulation:** The amplitude of a sinusoidal signal with fixed frequency and phase is varied in proportion to a given signal.

**Purpose:**
- Adaptation of the information signal to the transmission channel
- Shift of the information signal to an assigned frequency band

- Efficient antenna design: size is at least $1/4^{th}$ of signal wavelength
  $\Rightarrow$ antennas for lowpass signals would be too large ($f = 3$ kHz, $\lambda = 100,000$ m).

\[ j \Phi(\omega) \]
\[ jF(\omega) \]
Simultaneous transmission of several information signals (e.g. radio broadcasting)
4.2 Double-Sideband Suppressed Carrier AM (DSB-SC)

4.2.1 Modulation

Generation of DSB-SC modulated signal:

\[ \phi(t) = f(t) \cos(\omega_c t) \]

\( \phi(t) \): modulated transmit signal  
\( f(t) \): modulating signal, real valued  
\( \cos(\omega_c t) \): carrier signal, \( \omega_c \): carrier frequency in rad/sec

Spectrum of DSB-SC modulated signal:

\[ \Phi(\omega) = \frac{1}{2}F(\omega - \omega_c) + \frac{1}{2}F(\omega + \omega_c) \]
• Carrier frequency has to be larger than twice the bandwidth \( \omega \geq 2W \).

• Bandwidth of the modulated signal \( \phi(t) \) is twice as large as the bandwidth of the modulating signal \( f(t) \).

• No separate carrier is present in \( \phi(t) \).

• Upper sideband: spectral content for positive frequencies above \( \omega_c \).
  Lower sideband: spectral content for positive frequencies below \( \omega_c \).

• Information in upper and lower sideband are redundant since \( \Phi(\omega_c + \omega) = \Phi^*(\omega_c - \omega) \), or equivalently: \( |\Phi(\omega_c + \omega)| = |\Phi(\omega_c - \omega)| \) and \( \angle \Phi(\omega_c + \omega) = -\angle \Phi(\omega_c - \omega) \)
4.2.2 Demodulation

\[ \phi(t) = f(t) \cos(\omega_c t) \]

Before lowpass filtering:

\[ \phi(t) \, 2 \cos(\omega_c t) = 2f(t) \cos^2(\omega_c t) = f(t) (1 + \cos(2\omega_c t)) \]

\[ F\{\phi(t) \, 2 \cos(\omega_c t)\} = F(\omega) + \frac{1}{2}F(\omega - 2\omega_c) + \frac{1}{2}F(\omega + 2\omega_c) \]

After lowpass filtering:

\[ \hat{F}(\omega) = F(\omega) \]

The oscillators at the transmitter and receiver have to be synchronized, i.e. the carrier frequency \( \omega_c \) as well as the phase must be identical (coherent demodulation).
Influence of Frequency and Phase Offset:
The oscillator at the receiver has a constant phase offset of $\theta_0$ as well as a slightly different carrier frequency of $\omega_c + \Delta \omega$ when compared to the one at the transmitter.

Before lowpass filtering:
\[
\phi(t) 2 \cos((\omega_c + \Delta \omega) t + \theta_0) = 2f(t) \cos(\omega_c t) \cos((\omega_c + \Delta \omega) t + \theta_0)
\]
\[
= f(t) \cos((2\omega_c + \Delta \omega) t + \theta_0) + f(t) \cos(\Delta \omega t + \theta_0)
\]

After lowpass filtering:
\[
\hat{f}(t) = f(t) \cos(\Delta \omega t + \theta_0)
\]
\[
= \frac{1}{2} f(t) \exp(j\Delta \omega t) \exp(j\theta) + \frac{1}{2} f(t) \exp(-j\Delta \omega t) \exp(-j\theta)
\]
\[
\hat{F}(\omega) = \frac{1}{2} \exp(j\theta) F(\omega - \Delta \omega) + \frac{1}{2} \exp(-j\theta) F(\omega + \Delta \omega)
\]
Phase error only (i.e. $\Delta \omega = 0$):

$$\hat{f}(t) = f(t) \cos(\theta_0) \quad \circ \bullet \quad \hat{F}(\omega) = F(\omega) \cos(\theta_0)$$

$\Rightarrow$ The recovered signal is scaled by a constant. For $\theta_0 = \pm 90^\circ$ we have $\hat{f}(t) = 0$.

Frequency error only (i.e. $\theta_0 = 0$):

$$\hat{f}(t) = f(t) \cos(\Delta \omega t) \quad \circ \bullet \quad \hat{F}(\omega) = \frac{1}{2} F(\omega - \Delta \omega) + \frac{1}{2} F(\omega + \Delta \omega)$$

$\Rightarrow$ The recovered signal is still modulated by a cosine signal of low frequency $\Delta \omega$. 

Dr. Tanja Karp 10
Pilot Carrier

- send a sinusoidal tone whose frequency and phase is proportional to $\omega_c$

- sent outside the passband of the modulate signal

- Receiver detects the tone, translates to correct frequency (doubling) and demodulates
Example - Commercial Stereo FM Stations

Transmitter

- need to transmit left(L) and right(R) as well as (L+R) for monophonic
- (L+R) occupies $0 - 15\, kHz$
- so does (L-R), so shift up using DSB-SC with $\omega_c = 38\, kHz$
- place pilot tone at 19kHz
Receiver

- narrow bandpass filter at 19kHz and then double to 38kHz
- after demodulation using pilot tone, we have

\[
\begin{align*}
\text{Left channel} &= (L + R) + (L - R) = 2L \\
\text{Right channel} &= (L + R) - (L - R) = 2R
\end{align*}
\]
Phase Locked Loop (PLL)

- Pilot Tone Problem - BP filters drift in tuning, bad at rejecting noise
- Solution: Phase Locked Loop (PLL)

- Operation when Voltage Controlled Oscillator (VCO) frequency ($\omega_{VCO}$) is close to $\omega_c$
  - low-frequency component of output is proportional to magnitude and sign of phase difference
  - this voltage adjusts $\omega_{VCO}$ to keep phase difference a minimum

- Bandwidth of PLL determined by LPF
  - Small BW $\Rightarrow$ good noise rejection but receiver may never lock
  - Large BW $\Rightarrow$ good lock but bad noise rejection
4.3 Double-Sideband Large Carrier AM

4.3.1 Modulation

• Reduces complexity of receiver

• Since this type of AM is used in commercial broadcast stations, usually termed AM

• Similar to DSB-SC, except that we incorporate the carrier
  – carrier must be larger than the rest of the signal
  – ruins low-frequency response of the system, so must not require frequency response down to 0.

\[ \phi_{AM} = f(t) \cos(\omega_c t) + A \cos(\omega_c t) \]

\[ \Phi_{AM}(\omega) = \frac{1}{2} F(\omega + \omega_c) + \frac{1}{2} F(\omega - \omega_c) + \pi A \delta(\omega + \omega_c) + \pi A \delta(\omega - \omega_c) \]
• if $A$ is large enough signal recovery is done with envelope detection

$$[A + f(t)] \geq 0 \text{ for all } t$$
- Let $f(t) = \cos(\omega_m t)$, we define $m$ to control the amount of modulation

$$m = \frac{\text{peak DSB-SC amplitude}}{\text{peak carrier amplitude}}$$

$$\phi(t) = A \cos(\omega_c t) + mA \cos(\omega_m t) \cos(\omega_c t)$$

$$= A[1 + m \cos(\omega_m t)] \cos(\omega_c t)$$

- percentage of modulation for DSB-LC signal with sinusoidal modulation

$$\%_{\text{mod}} = \frac{A(1 + m) - A(1 - m)}{A(1 + m) + A(1 - m)} \times 100\% = m \times 100\%$$

- we call $m$ the modulation index

- in order to detect the signal with no distortion we require $m \leq 1$
4.3.2 Carrier and Sideband Power in AM

- carrier provides no information so it is just wasted power
- for an AM signal $\phi_{AM}(t) = A \cos(\omega_c t) + f(t) \cos(\omega_c t)$ the power is
  \[
  \phi_{AM}^2(t) = A^2 \cos^2(\omega_c t) + f^2(t) \cos^2(\omega_c t) + 2Af(t) \cos^2(\omega_c t)
  \]
  \[
  = A^2 \cos^2(\omega_c t) + f^2(t) \cos^2(\omega_c t)
  \]
  \[
  = A^2/2 + f^2(t)/2
  \]

- so we can express the total power as,
  \[
  P_t = P_c + P_s = \frac{1}{2}A^2 + \frac{1}{2}f^2(t)
  \]
  so that the fraction of the total power contained in the sidebands is
  \[
  \mu = \frac{P_s}{P_t} = \frac{f^2(t)}{A^2 + f^2(t)}
  \]
• so when \( f(t) = \cos(\omega_m t) \) we get

\[
\phi_{AM}^2(t) = \frac{1}{2}A^2 + \left(\frac{1}{2}\right)\frac{1}{2}m^2 A^2
\]

\[
\mu = \frac{m^2}{2 + m^2}
\]

• so for best case, i.e., \( m = 1 \), 67% of the total power is wasted with the carrier

### 4.3.3 Demodulation

• the price we pay for wasted power is a tradeoff for simple receiver design

• receiver is simply an envelope detector

![Diagram of envelope detector](image)
4.4 Quadrature AM (IQ)

- for real signal $f(t)$, $F(\omega) = F^*(-\omega)$
- using this symmetry we can transmit two signals that form a complex signal with same bandwidth
- we use two sinusoidal carriers, each exactly $90^\circ$ out of phase
  
  remember, $e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$
- transmitted over the same frequency band,
\[
\phi(t) = f(t) \cos(\omega_c t) + g(t) \sin(\omega_c t)
\]
\[
\phi(t) \cdot \cos(\omega_c t) = f(t) \cos^2(\omega_c t) + g(t) \sin(\omega_c t) \cos(\omega_c t)
\]
\[
= \frac{1}{2} f(t) + \frac{1}{2} f(t) \cos(2\omega_c t) + \frac{1}{2} f(t) \sin(2\omega_c t)
\]
\[
\phi(t) \cdot \sin(\omega_c t) = f(t) \cos(\omega_c t) \sin(\omega_c t) + g(t) \sin^2(\omega_c t)
\]
\[
= \frac{1}{2} f(t) \sin(2\omega_c t) + \frac{1}{2} g(t) - \frac{1}{2} \cos(2\omega_c t)
\]
4.5 Single-Sideband AM (SSB)

- remember for real $f(t)$, $F(-\omega) = F^*(\omega)$
- a single sideband contains entire information of the signal
- let’s just transmit the upper/lower sideband.
4.5.1 Modulation

- one way is to generate DSB signal, and then suppress one sideband with filtering
- hard to do in practice, can’t get ideal filters
- assume no low-frequency information \(\Rightarrow\) no components around \(\omega_c\)
- use heterodyning (frequency shifting), only need to design on sideband filter
- another way is the use of phasing
- assume a complex, single-frequency signal, \(f(t) = e^{j\omega_m t}\) with carrier signal \(f(t) = e^{j\omega_c t}\)
- multiplying we get \(\phi(t) = f(t) e^{j\omega_c t} = e^{j\omega_m t} e^{j\omega_c t}\)
- using the frequency-translation property of the Fourier Transform, our spectrum becomes

\[
\Phi(\omega) = 2\pi \delta(\omega - (\omega_c + \omega_m))
\]
• to make the signal $\phi(t)$ realizable, we take the $\mathbb{R}\{\phi(t)\}$

\[
\mathbb{R}\{\phi(t)\} = \mathbb{R}\{e^{j\omega_m t}\} \mathbb{R}\{e^{j\omega_c t}\} - \mathbb{I}\{e^{j\omega_m t}\} \mathbb{I}\{e^{j\omega_c t}\}
\]

\[
= \cos(\omega_m t) \cos(\omega_c t) - \sin(\omega_m t) \sin(\omega_c t)
\]

• So the upper side band is

\[
\phi_{SSB^+}(t) = \cos(\omega_m t) \cos(\omega_c t) - \sin(\omega_m t) \sin(\omega_c t)
\]
• likewise the lower sideband is
\[ \phi_{SSB_-}(t) = \cos(\omega_m t) \cos(\omega_c t) + \sin(\omega_m t) \sin(\omega_c t) \]

• in general we write,
\[ \phi_{SSB_\mp}(t) = f(t) \cos(\omega_c t) \pm \hat{f}(t) \sin(\omega_c t) \]

where \( \hat{f}(t) \) is \( f(t) \) shifted by 90°
4.5.2 Demodulation

Synchronous detection, analogous to DSB-SC

Influence of Frequency and Phase Offset:
The oscillator at the receiver has a constant phase offset of $\theta$ as well as a slightly different carrier frequency offset of $\Delta \omega$ giving

$$\phi_d(t) = \cos[(\omega_c + \Delta \omega)t + \theta]$$
Before lowpass filtering:

\[
\phi_{SSB \mp}(t) \phi_d(t) = [f(t) \cos(\omega_c t) \pm \hat{f}(t) \sin(\omega_c t)] \cos[(\omega_c + \Delta \omega)t + \theta]
\]

\[
= \frac{1}{2} f(t) \{\cos[(\Delta \omega)t + \theta] + \cos[(2\omega_c + \Delta \omega)t + \theta]\}
\]

\[
= \pm \frac{1}{2} \hat{f}(t) \{\sin[(\Delta \omega)t + \theta] - \sin[(2\omega_c + \Delta \omega)t + \theta]\}
\]

After lowpass filtering:

\[
e_o(t) = \frac{1}{2} f(t) \cos[(\Delta \omega)t + \theta] \mp \frac{1}{2} \hat{f}(t) \sin[(\Delta \omega)t + \theta]
\]

Phase error only (i.e. \(\Delta \omega = 0\)):

\[
e_o(t) = \frac{1}{2} [f(t) \cos \theta \mp \hat{f}(t) \sin \theta]
\]

To understand this better we re-write the above equation as

\[
e_o(t) = \frac{1}{2} \Re\{[f(t) \pm j\hat{f}(t)]e^{j\theta}\}
\]

⇒ So phase error in the receiver oscillator results in phase distortion.
Frequency error only (i.e. $\theta = 0$):

$$e_0(t) = \frac{1}{2} [f(t) \cos(\Delta \omega t) \mp \hat{f}(t) \sin(\Delta \omega t)]$$

or

$$e_o(t) = \frac{1}{2} \Re\{[f(t) \pm j\hat{f}(t)] e^{j\Delta \omega t}\}$$

⇒ Demodulated signal contains spectral shifts and phase distortions.
4.6 Vestigial-Sideband AM (VSB)

- compromise between DSB and SSB.
- partial suppression of one sideband

\[
\Phi_{VSB}(\omega) = \frac{1}{2}F(\omega - \omega_c) + \frac{1}{2}F(\omega + \omega_c)\] \(H_{V}(\omega)\)

- after synchronous detection we have

\[
E_o(\omega) = \frac{1}{4}F(\omega)H_{V}(\omega + \omega_c) + \frac{1}{4}F(\omega)H_{V}(\omega - \omega_c)
\]

\[
= \frac{1}{4}F(\omega)[H_{V}(\omega + \omega_c) + H_{V}(\omega - \omega_c)]
\]
thus for reproduction of $f(t)$ we require

$$[H_V(\omega - \omega_c) + H_V(\omega + \omega_c)]_{LP} = \text{constant}$$

- magnitude can be satisfied, but phase requirements are hard to satisfy
- use when phase is not important
4.6.1 Video Transmission in Commercial TV Systems

- video requires $4 \text{MHz}$ bandwidth to transmit
- so DSB would require $8 \text{MHz}$ per channel
- use VSB to decrease the needed bandwidth to $5 \text{MHz}$
4.7 Summary

Double Sideband-Suppressed Carrier (DSB-SC)

- spectrum at $\omega_c$ is a copy of baseband spectrum with scaling factor of 1/2
- information is sidebands is redundant
- for coherent detection, we must have same frequency and phase of carrier signal
- detection can be done with pilot tone, PLL
Double Sideband-Large Carrier (DSB-LC)

- same as DSB-SC, with an addition of a carrier term
- detection is a simple envelope detector
- wastes, at best case, 67% of the power in the carrier term
- frequency response at low-frequencies are ruined

Quadrature Amplitude Modulation (QAM)

- efficient utilization of bandwidth
• forms a complex signal with two sinusoidal carriers of same frequency, $90^\circ$ out of phase

**Single Sideband Modulation (SSB)**

- suppress either upper or lower sideband for more efficient bandwidth utilization
- generated by filtering DSB-SC
• can also use phasing to cancel the “negative” frequencies
• can use either suppressed carrier, pilot tone, or large carrier AM also

Vestigial Sideband (VSB)

• compromises DSB and SSB
• transmitter and receiver filters must be complementary, i.e., they must add to a constant at baseband
• phase must not be important