Amplitude Modulation – Part 1

Instructor: Prof. Farshad Lahouti

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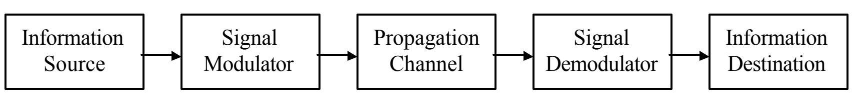
Analog Communication System



- Analog signals may be transmitted directly via carrier modulation over the propagation channel and to be carrier-demodulated at the receiver.

Transmitter → Modulator

Receiver → Demodulator

Modulation: The process by which some characteristics of a carrier signal (i.e. modulated signal) is varied in accordance with message signal (i.e. modulating signal)

- $f(t)$: message signal

A bandlimited signal whose frequency content is in the neighbourhood of f=0 (DC) $\Rightarrow$ baseband signal

- $c(t)$: the carrier signal, independent of $f(t)$

$$c(t) = A_c \cos(\omega_c t + \theta_c)$$

A_c : Carrier amplitude

f_c : Carrier frequency, $\omega_c = 2\pi f_c$ (radian frequency)

θ_c : Carrier phase

$f(t)$ modulates $c(t)$ in either amplitude, frequency or phase. In effect, modulation converts $f(t)$ to a bandpass form, in the neighborhood of the center frequency f_c.

Why is Modulation Required?

- **<u>To achieve easy radiation:</u>** If the communication channel consists of free space, antennas are required to radiate and receive the signal. Dimension of the antennas is limited by the corresponding wavelength.

Example: Voice signal bandwidth f=3kHz

$$\lambda = \frac{c}{f} = \frac{3 \cdot 10^8}{3 \cdot 10^3} = 10^5 \text{m}$$

$$\Rightarrow \lambda/4 = 25000 \text{ m}$$

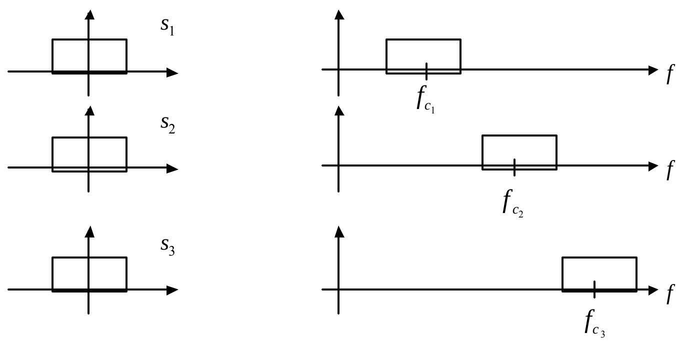
If we modulate a carrier wave @ f_c = 100MHz with the voice signal

$$\lambda = \frac{c}{f} = \frac{3 \cdot 10^8}{100 \cdot 10^6} = 3 \text{ m}$$

$$\Rightarrow \lambda/4 = 75 \text{ cm}$$

Why is Modulation Required? (Cont'd)

- **<u>To accommodate for simultaneous transmission of several signals</u>**



Example: Radio/TV broadcasting

Why is Modulation Required? (Cont'd)

- To expand the bandwidth of the transmitted signal for better transmission quality (to reduce noise and interference)

$$C = B \cdot \log_2(1 + SNR)$$

C → Channel capacity; B → Bandwidth; SNR → Signal-to-noise ratio

Channel capacity: Maximum achievable information rate that can be transmitted over the channel

$$SNR = 2^{\frac{C}{B}} - 1$$

$B \uparrow$ The required SNR (for fixed noise level, corresponding signal power) decreases

Amplitude Modulation (AM)

(Ch. 5 in Textbook)

Objectives:

- To study different amplitude modulation scheme
- To study generation and detection of AM signals
- To study application of AM

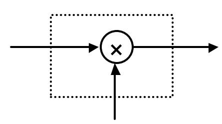
We will study

- **Double Sideband Suppressed Carrier (DSB-SC) Modulation**
- **Double Sideband Large Carrier (DSB-LC) Modulation:** Commercial broadcast stations use this type and it is commonly known as just amplitude modulation (AM).
- **Single Sideband (SSB) Modulation**
- **Vestigial Sideband (VSB) Modulation**

Double Side Band Supressed Carrier (DSB-SC)

(5.1 in Textbook)

$f(t)$



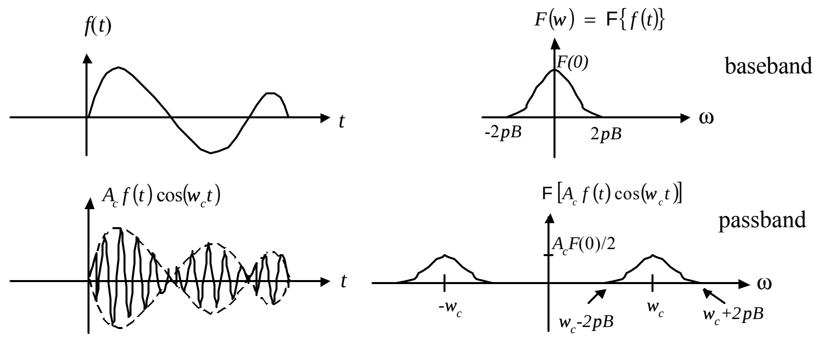
$\boldsymbol{f}(t) = A_c f(t)\cos(w_c t)$

$c(t) = A_c \cos(w_c t)$

$$F(w) = \mathcal{F}\{f(t)\} \quad \mathbf{F}(w) = \mathcal{F}\{\boldsymbol{f}(t)\}$$

$$\begin{aligned}\mathbf{F}(w) &= \mathcal{F}[A_c f(t)\cos(w_c t)] \\ &= \mathcal{F}\left(\frac{A_c}{2} f(t) e^{jw_c t} + \frac{A_c}{2} f(t) e^{-jw_c t}\right) \\ &= \frac{A_c}{2} F(w - w_c) + \frac{A_c}{2} F(w + w_c)\end{aligned}$$

$$\cos w_c t = \frac{1}{2}\left(e^{jw_c t} + e^{-jw_c t}\right)$$



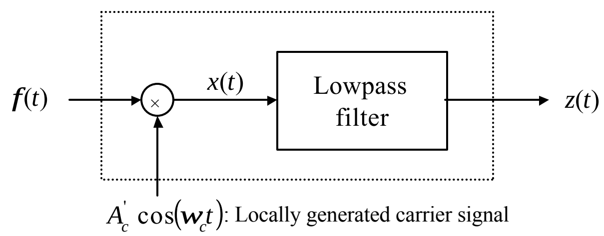
Observations:

• $f(t)$ undergoes a phase reversal whenever $f(t)$ crosses zero. The envelope of $f(t)$ is different from $f(t)$. Both amplitude and phase of $f(t)$ carry information of $f(t)$.

• The transmission bandwidth required by DSB-SC is b_T=2B.

Demodulation of DSB-SC signals

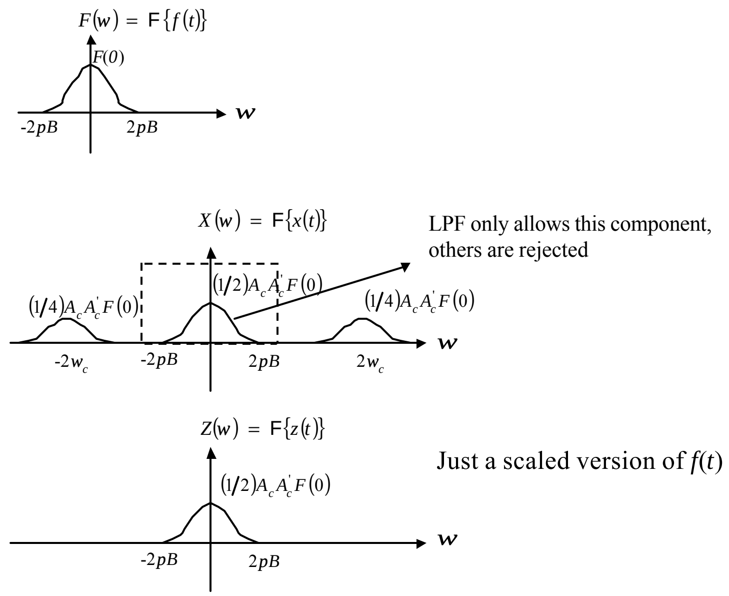
Given $f(t)$, how will be the message signal $f(t)$ be recovered?



$$x(t) = f(t)A_c' \cos(w_c t) = A_c' A_c f(t) \cos^2(w_c t)$$
$$= \frac{1}{2} A_c' A_c f(t) + \frac{1}{2} A_c' A_c f(t) \cos(2 w_c t)$$

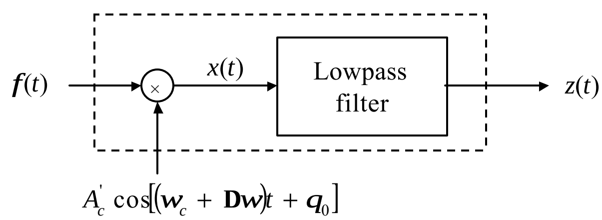
Let $X(w) = \mathcal{F}\{x(t)\}$

$$X(w) = \frac{1}{2} A_c A_c' F(w) + \frac{1}{4} A_c A_c' [F(w - 2w_c) + F(w + 2w_c)]$$



Effect of error in phase and frequency at the receiver

Now assume a frequency error and a phase error in the locally generated signal at the receiver.



$$x(t) = f(t)A_c' \cos[(\omega_c + \Delta\omega)t + \theta_0]$$

$$= A_c A_c' f(t)\cos(\omega_c t)\cos[(\omega_c + \Delta\omega)t + \theta_0]$$

$$= \underbrace{\frac{1}{2}A_c A_c' f(t)\cos(\Delta\omega t + \theta_0)}_{\text{Only this term goes through LPF}} + \frac{1}{2}A_c A_c' f(t)\cos[(2\omega_c + \Delta\omega)t + \theta_0]$$

$$z(t) = \frac{1}{2} A_c A_c' f(t) \cos(\Delta\omega t + \theta_0)$$

- If $\Delta\omega$=0 and θ_0= 0, the output is $z(t) = \frac{1}{2} A_c A_c' f(t)$ ⇒ no distortion
- If $\Delta\omega$ =0, the output is $z(t) = \frac{1}{2} A_c A_c' f(t) \cos(\theta_0)$

  The phase error introduces a variable attenuation factor. For small fixed phase errors, this is quite tolerable. If θ_0= ±90º, the received signal is wiped out.

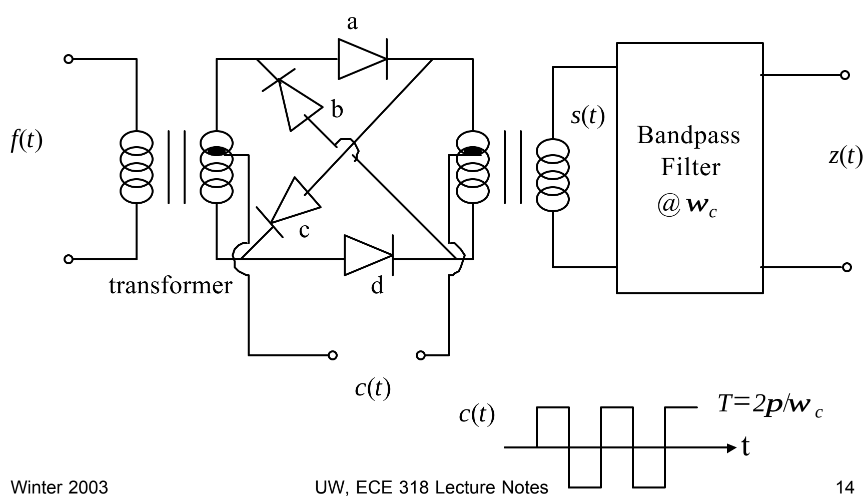
- If θ_0= 0, the output is $z(t) = \frac{1}{2} A_c A_c' f(t) \cos(\Delta\omega t)$

To recover $f(t)$ accurately from $f(t)$, we need to use a synchronized oscillator *Coherent detection* (synchronous detection) is required.

Generation of DSB-SC signals: *A practical implementation*

Circuitry aspects of this topic will not be in any examination, mathematical aspects may be tested.

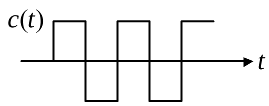
Ring Modulator



The diodes are controlled by a square-wave carrier $c(t)$ of frequency w_c, assuming $|c(t)|>>|f(t)|$

$c(t) > 0$ Diodes a and d conduct $\quad s(t) = f(t)$

$c(t) < 0$ Diodes b and c conduct $\quad s(t) = -f(t)$



Expressing in terms of Fourier Series

$$c(t) = \frac{4}{p}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos[w_c t(2n-1)]$$

$$s(t) = c(t)f(t)$$

$$s(t) = f(t)\frac{4}{p}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos[w_c t(2n-1)]$$

Output of BPF $\quad z(t) = \frac{4}{p} f(t)\cos(w_c t)$

Demodulation of DSB-SC Signals: *A Practical Implementation*

Costas Receiver

The content of this page will not be asked in any examination.