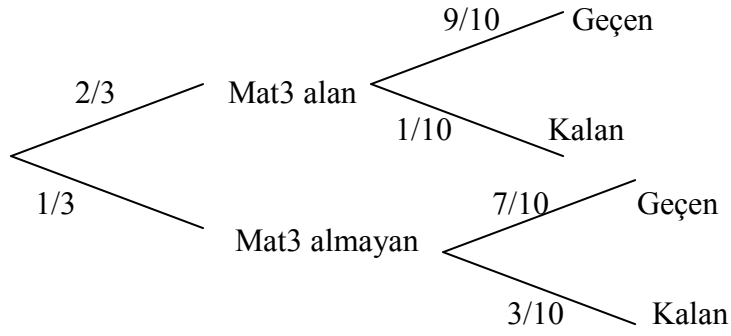


1-



$$P(\text{Mat3 almamak, başarısızlık}) = \frac{\frac{1}{3} * \frac{3}{10}}{\frac{2}{3} * \frac{1}{10} + \frac{1}{3} * \frac{3}{10}} = \frac{3}{5}$$

2-

$$\mu = 60$$

$$\sigma = 10$$

$$z = \frac{x - \mu}{\sigma} = \frac{x - 60}{10}$$

0,4 alanı veren $z=1,28$

$$1,28 = \frac{x - 60}{10}$$

$$x = 72,8$$

AA almak için gereken minumum puan 73 olmalıdır.

3-

$$k = ?$$

$$\int_{x=1}^2 \int_{y=0}^2 k(0,5x + y) dy dx = 1 \quad \text{olmalı,}$$

$$\int_{x=1}^2 \int_{y=0}^2 k(0,5x + y) dy dx = k \int_{x=1}^2 \left(0,5xy + \frac{y^2}{2} \right) \Big|_{y=0}^2 dx = k \int_{x=1}^2 (x + 2) dx = k \left(\frac{x^2}{2} + 2x \right) \Big|_{x=1}^2 = \frac{7}{2} k$$

Buradan,

$$\frac{7}{2}k = 1 \quad \Rightarrow \quad \boxed{k = \frac{2}{7}}$$

3-a)

$$\begin{aligned} P(X > 1, Y < 1) &= \int_{x=1}^2 \int_{y=0}^1 \frac{2}{7} (0,5x + y) dx dy = \frac{2}{7} \int_{x=1}^2 \left(0,5xy + \frac{y^2}{2} \right) \Big|_{y=0}^1 dx = \frac{2}{7} \int_{x=1}^2 (0,5x + 0,5) dx \\ &= \frac{2}{7} \left(0,5 \frac{x^2}{2} + \frac{x}{2} \right) \Big|_{x=1}^2 = 0,357 \end{aligned}$$

3-b)

$$\text{Var}(X) = E(x^2) - [E(x)]^2$$

$$\begin{aligned} E(x) &= \frac{2}{7} \int_{x=1}^2 \int_{y=0}^2 x(0,5x + y) dy dx = \frac{2}{7} \int_{x=1}^2 \int_{y=0}^2 (0,5x^2 + xy) dx dy = \frac{2}{7} \int_{x=1}^2 \left(0,5x^2 y + \frac{xy^2}{2} \right) \Big|_{y=0}^2 dx \\ &= \frac{2}{7} \int_{x=1}^2 (x^2 + 2x) dx = \frac{2}{7} \left(\frac{x^3}{3} + x^2 \right) \Big|_{x=1}^2 = \frac{32}{21} = 1,524 \end{aligned}$$

$$\begin{aligned} E(x^2) &= \frac{2}{7} \int_{x=1}^2 \int_{y=0}^2 x^2 (0,5x + y) dy dx = \frac{2}{7} \int_{x=1}^2 (0,5x^3 + x^2 y) dy dx = \frac{2}{7} \int_{x=1}^2 \left(0,5x^3 y + \frac{x^2 y^2}{2} \right) \Big|_{y=0}^2 dx \\ &= \frac{2}{7} \int_{x=1}^2 (x^3 + 2x^2) dx = \frac{2}{7} \left(\frac{x^4}{4} + \frac{2x^3}{3} \right) \Big|_{x=1}^2 = \frac{2}{7} \left(4 + \frac{16}{3} - \frac{1}{4} - \frac{2}{3} \right) = 2,405 \end{aligned}$$

$$\text{Var}(X) = E(x^2) - [E(x)]^2 = 2,405 - (1,524)^2 = 0,0824$$

3-c)

$$\rho = \frac{\sigma_{xy}}{\sigma_x \sigma_y} = \frac{\text{COV}}{\sigma_x \sigma_y}$$

$$\sigma_x = \sqrt{\text{Var}(x)} = \sqrt{0,0824} = 0,287$$

$$\text{Var}(y) = E(y^2) - [E(y)]^2$$

$$\sigma_y = \sqrt{\text{Var}(y)}$$

$$E(y) = \frac{2}{7} \int_{x=1}^2 \int_{y=0}^2 y(0,5x + y) dy dx = \frac{2}{7} \int_{x=1}^2 \int_{y=0}^2 (0,5xy + y^2) dy dx = \frac{2}{7} \int_{x=1}^2 0,5 \frac{xy^2}{2} + \frac{y^3}{3} \Big|_{y=0}^2 dx$$

$$= \frac{2}{7} \int_{x=1}^2 (x + \frac{8}{3}) dx = \frac{2}{7} \left(\frac{x^2}{2} + \frac{8}{3}x \right) \Big|_1^2 = \frac{2}{7} (2 + \frac{16}{3} - \frac{1}{2} - \frac{8}{3}) = 1,190$$

$$E(y^2) = \frac{2}{7} \int_{x=1}^2 \int_{y=0}^2 y^2(0,5x + y) dy dx = \frac{2}{7} \int_{x=1}^2 \int_{y=0}^2 (0,5xy^2 + y^3) dy dx$$

$$= \frac{2}{7} \int_{x=1}^2 \left(\frac{0,5xy^3}{3} + \frac{y^4}{4} \right) \Big|_{y=0}^2 dx = \frac{2}{7} \int_{x=1}^2 \left(\frac{4}{3}x + 4 \right) dx = \frac{2}{7} \left(\frac{4}{3} \frac{x^2}{2} + 4x \right) \Big|_{x=1}^2$$

$$= \frac{2}{7} \left(\frac{4}{3} \frac{4}{2} + 8 - \frac{4}{6} - 4 \right) = 1,714$$

$$\begin{aligned} \text{Var}(y) &= E(y^2) - [E(y)]^2 \\ &= 1,714 - (1,190)^2 = 0,298 \end{aligned}$$

$$\sigma_y = \sqrt{\text{Var}(y)} = \sqrt{0,298} = 0,546$$

$$\sigma_{xy} = \text{cov aryans} = E(xy) - \mu_x \mu_y = E(xy) - E(x)E(y)$$

$$E(xy) = \frac{2}{7} \int_{x=1}^2 \int_{y=0}^2 xy(0,5x + y) dy dx = \frac{2}{7} \int_{x=1}^2 \int_{y=0}^2 (0,5x^2y + xy^2) dy dx$$

$$= \frac{2}{7} \int_{x=1}^2 \left(\frac{0,5x^2y^2}{2} + \frac{xy^3}{3} \right) \Big|_{y=0}^2 dx = \frac{2}{7} \int_{x=1}^2 \left(x^2 + \frac{8}{3}x \right) dx = \frac{2}{7} \left(\frac{x^3}{3} + \frac{8}{6}x^2 \right) \Big|_{x=1}^2$$

$$= \frac{2}{7} \left(\frac{8}{3} + \frac{16}{3} - \frac{1}{3} - \frac{4}{3} \right) = 1,81$$

$$\begin{aligned} \sigma_{xy} &= \text{cov aryans} = E(xy) - \mu_x \mu_y = E(xy) - E(x)E(y) \\ &= 1,81 - 1,524 * 1,19 = -0,00356 \end{aligned}$$

$$\rho = \frac{\sigma_{xy}}{\sigma_x \sigma_y} = \frac{\text{cov}}{\sigma_x \sigma_y} = \frac{-0,00356}{0,287 * 0,546} = -0,227$$

4-

$$\sum y = a.n + b \sum x$$

$$y = a + bx$$

$$\sum xy = a. \sum x + b \sum x^2$$

Ayran (y)	Sıcaklık (x)	x^2	xy
25	7	49	175
37	12	144	444
50	20	400	1000
70	25	625	1750
80	30	900	2400
90	34	1156	3060
+			
352	128	3274	8829

$$(I) 352 = 6a + 128b$$

$$(II) 8829 = 128a + 3274b$$

(I). denklemi -21,33 ile çarpıp (II). denklemle taraf tarafa toplayalım.

$$-7509,33 = -128a - 2730,24b$$

$$8829 = 128a + 3274b$$

$$+$$

$$1319,67 = 543,76b$$

$$b = 2,43$$

$$a = \frac{352 - 128 * 2,43}{6} = 6,83$$

$$y = a + bx$$

$$y = 6,83 + 2,43x$$

$$x = 40$$

$$y = 6,83 + 2,43 * 40 = 104$$

Sıcaklık 40⁰ C olduğunda 104 kutu ayran tüketilmesi beklenir.